

Dynamical radion superfield in 5D action

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Abstract

We derive 5D $\mathcal{N} = 1$ superspace action including the radion superfield. The radion is treated as a dynamical field and identified as a solution of the equation of motion even in the presence of the radius stabilization mechanism. Our derivation is systematic and based on the superconformal formulation of 5D supergravity. We can read off the couplings of the dynamical radion superfield to the matter superfields from our result. The correct radion mass can be obtained by calculating the radion potential from our superspace action.

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1 Introduction

Five dimensional supergravity (5D SUGRA) compactified on an orbifold S^1/Z_2 has been thoroughly investigated since it is shown to appear as an effective theory of the strongly-coupled heterotic string theory [1] compactified on a Calabi-Yau 3-fold [2]. Especially, the Randall-Sundrum model [3] is attractive as an alternative solution to the hierarchy problem, and a huge number of researches on this model have been done. In this model, the background geometry is a slice of the anti-de Sitter (AdS) spacetime and the metric has the form of ¹

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu = e^{-2ky} \eta_{mn} dx^m dx^n - dy^2 \\ &= e^{-2kR\vartheta} \eta_{mn} dx^m dx^n - R^2 d\vartheta^2, \end{aligned} \quad (1.1)$$

where k is the AdS curvature and R is the radius of the orbifold. The physical range of the extra space is $0 \leq y \leq \pi R$. In the second line, we have changed the coordinate y to the dimensionless coordinate $\vartheta \equiv y/R$.

In such a brane-world model, the radius of the compactified extra dimension is generically a dynamical degree of freedom, *the radion*. In the original Randall-Sundrum model [3], the radius of the orbifold is undetermined by the dynamics and thus the radion is a massless field. Hence, it remains to be a dynamical degree of freedom in low energies and should be taken into account in 4D effective theory. A naive way of introducing the radion mode into the theory is to promote the radius R to a 4D field $r(x)$, that is, to consider the metric of the form [4, 5]

$$ds^2 = e^{-2kr(x)\vartheta} g_{mn}^{(4)}(x) dx^m dx^n - r^2(x) d\vartheta^2, \quad (1.2)$$

where $g_{mn}^{(4)}$ is the 4D graviton. However, this is not a solution of the Einstein equation, even at the linearized order. This means that the radion mode defined here is not a mass eigenstate and has mixings with the massive Kaluza-Klein (K.K.) modes which we have dropped in Eq.(1.2). Thus, such K.K. modes cannot simply be dropped when they are integrated out. Therefore, a naive ansatz (1.2) need to be corrected. Furthermore, this metric means the radion $r(x)$ does not couple to the brane at $\vartheta = 0$, which contradicts the fact that it couples to the boundary branes like a Brans-Dicke scalar [5, 6]. In order to treat the radion as a dynamical field, we have to define it as a solution of the equation of motion. Then, we can safely drop the K.K. modes after we integrate them out. Such treatment for the radion is discussed in Ref. [7] in the absence of the radius stabilization mechanism. In that case, the physical gravitational modes are the 4D graviton, its K.K. modes, and the radion mode. Note that there is no K.K. tower above the massless radion. All such K.K. modes can be gauged away and thus are unphysical. The authors of Ref. [7] found a gauge where the radion mode is contained in the metric as

$$ds^2 = e^{-2ky + \tilde{b}(x)e^{2ky}} g_{mn}^{(4)}(x) dx^m dx^n - \left(1 - \tilde{b}(x)e^{2ky}\right)^2 dy^2, \quad (1.3)$$

where $\tilde{b}(x)$ is a 4D massless field that corresponds to the radion fluctuation mode around the background value. In contrast to the naive ansatz (1.2), this certainly satisfies the

¹Throughout this paper, we will use $\mu, \nu, \dots = 0, 1, 2, 3, 4$ for the 5D world vector indices, and $m, n, \dots = 0, 1, 2, 3$ for the 4D indices. The coordinate of the extra dimension is denoted as $y \equiv x^4$.

linearized Einstein equation. (Note that the mode function of $\tilde{b}(x)$ is the correct one, e^{2ky} .) This gauge is useful because the whole spacetime is covered by one coordinate patch and simultaneously the 4D boundary planes are expressed by constant values of y .²

In order to construct a realistic model, we have to introduce some stabilization mechanism for the radius R . One of the main stabilization mechanism is proposed in Ref. [9], and it involves a bulk scalar field that has a nontrivial vacuum configuration. In such a case, the metric receives the backreaction from the bulk scalar configuration, and the background geometry deviates from the AdS spacetime. The radion dynamics in this case is thoroughly investigated in Ref. [10]. In this case, the radion mode resides not only in the metric but also in the bulk scalar field that is relevant to the radius stabilization.

The supersymmetric extension of the Randall-Sundrum model has also been investigated in many papers [11]. In this case, the radion belongs to an $\mathcal{N} = 1$ chiral multiplet in 4D effective theory. The corresponding radion superfield is identified in Ref. [8, 12] in the absence of the bulk matter fields. The couplings between the radion superfield and the bulk matter superfields are provided in Ref. [13], but their derivation is based on the naive ansatz (1.2) and should be corrected. There are also works that try to identify the radion multiplet in the context of the superconformal gravity [14, 15]. In Ref. [14], a chiral multiplet that contains the extra component of the fünfbein e_y^4 is constructed from component fields of 5D superconformal multiplets. The authors of Ref. [15] have clarified the appearance of such a chiral multiplet, which is usually called the radion multiplet, in the $\mathcal{N} = 1$ description of the superconformal gravity action. However, this multiplet is not the radion multiplet itself although it is closely related to the latter, because e_y^4 is a 5D field while the radion is a 4D field. In order to clarify the relation between them, we have to solve the equations of motion. The purpose of this paper is to derive 5D action written by $\mathcal{N} = 1$ superfields including the dynamical radion superfield defined as a solution of the equations of motion, in an appropriate way. Roughly speaking, our work corresponds to an extension of Ref. [8] to the case where the bulk and boundary matters and the radius stabilization mechanism exist.

In our previous paper [16], we have derived 5D superspace action on a general warped background directly from 5D SUGRA action.³ In this work, however, we have fixed the gravitational multiplet to its background value, and dropped all the fluctuation modes including the radion. Thus, we will derive the desired 5D action by introducing the dynamical radion mode in the 5D superspace action obtained in our previous work.

The paper is organized as follows. In the next section, we will briefly review the discussion in Ref. [10] to understand the situation, and explicitly identify the radion mode as a solution of the linearized equation of motion in the model of Ref. [17] as an example. In Section 3, we will derive the desired superspace action, and clarify the couplings of the radion superfield to the matter superfields. Section 4 is devoted to the summary. We collect the equations held by the classical background solution in Appendix A, and give

²For example, the Gaussian normal coordinates, which is convenient to deal with the junction conditions at the boundaries, need two coordinate patches to cover the whole spacetime, and in the Newton gauge, which is convenient to discuss the bulk dynamics, the boundary planes are no longer expressed by constant values of y .

³The same superspace action is also obtained in Ref. [15] independently of our work, at the stage before the superconformal gauge fixing.

some comment on the Newton gauge in Appendix B. In Appendix C, we collect explicit forms of the superfields in terms of the superconformal notation of Ref. [14, 18].

2 Dynamical radion mode

In this section, we will identify the dynamical radion mode in the presence of the stabilization mechanism. Some results of this section are contained in Refs. [10, 17, 19]. We will provide a self-contained review before deriving the desired superspace action since it is helpful to understand the situation around the radion.

2.1 Linearized equations of motion

First, we will derive the linearized equations of motion in order to discuss the dynamical radion mode.

For simplicity, we will assume that only one bulk scalar field has a nontrivial vacuum configuration in the following. Then, the lagrangian is ⁴

$$\mathcal{L} = -M_5^3 \sqrt{g} \mathcal{R} + \sqrt{g} \left\{ \frac{1}{2} \partial^\mu \bar{\varphi} \partial_\mu \varphi - V(\varphi) \right\} + \sqrt{|g_0|} \lambda_0(\varphi) \delta(y) + \sqrt{|g_\pi|} \lambda_\pi(\varphi) \delta(y - \pi R) + \cdots, \quad (2.1)$$

where M_5 is the 5D Planck mass, $g \equiv \det(g_{\mu\nu})$ and \mathcal{R} is the 5D Ricci scalar. $V(\varphi)$ and $\lambda_0(\varphi)$, $\lambda_\pi(\varphi)$ are the scalar potentials in the bulk and on the boundaries respectively, and g_0 and g_π are the determinants of the induced metrics on the branes. The ellipsis denotes terms irrelevant to the radius stabilization.

The background that preserves 4D Poincaré invariance is

$$ds^2 = e^{2\sigma(y)} \eta_{mn} dx^m dx^n - dy^2, \\ \varphi = \varphi_{\text{cl}}(y). \quad (2.2)$$

The equations of motion and the jump conditions for this background are listed in Appendix A.

In order to discuss the dynamical degrees of freedom, we should solve the linearized 5D Einstein equation and the field equations. For this purpose, the Newton gauge is useful [19, 20]. In this gauge, the fluctuation modes around the background are parametrized as

$$ds^2 = e^{2\sigma} (\eta_{mn} + h_{mn}^{\text{TT}} + 2B\eta_{mn}) dx^m dx^n - (1 - 4B) dy^2, \\ \varphi = \varphi_{\text{cl}} + \tilde{\varphi}, \quad (2.3)$$

where h_{mn}^{TT} is the transverse traceless mode, *i.e.*, $\partial^m h_{mn}^{\text{TT}} = 0$, $\eta^{mn} h_{mn}^{\text{TT}} = 0$. The linearized

⁴The metric convention is $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1, -1)$.

Einstein equations in this gauge are written as follows.

$$\{e^{-2\sigma}\square_4 - \partial_y^2 - 4\dot{\sigma}\partial_y\} h_{mn}^{\text{TT}} = 0, \quad (2.4)$$

$$\begin{aligned} & \{e^{-2\sigma}\square_4 - \partial_y^2 - 10\dot{\sigma}\partial_y - 4(\ddot{\sigma} + 4\dot{\sigma}^2)\} B \\ &= \frac{4}{3}\kappa^3 \text{Re} \left\{ \frac{\partial V}{\partial \varphi}(\varphi_{\text{cl}})\tilde{\varphi} + \frac{1}{2} \sum_{\vartheta^*=0,\pi} \frac{\partial \lambda_{\vartheta^*}}{\partial \varphi}(\varphi_{\text{cl}})\tilde{\varphi} \cdot \delta(y - R\vartheta^*) \right\}, \end{aligned} \quad (2.5)$$

$$(\partial_y + 2\dot{\sigma}) B = -\frac{2}{3}\kappa^3 \text{Re}(\dot{\varphi}_{\text{cl}}\tilde{\varphi}), \quad (2.6)$$

where $\square_4 \equiv \eta^{mn}\partial_m\partial_n$, $\kappa \equiv 1/M_5$ and the dot denotes the derivative with respect to y . The first equation comes from the traceless part of (m, n) -component of the linearized Einstein equation, the second one from the trace part of (m, n) -component and the last one from (m, y) -component. The equation from (y, y) -component is not shown because it is not independent of the above equations. Here, we have used Eq.(A.1).

From Eqs.(2.4) and (2.5), we can obtain the following boundary conditions.

$$\partial_y h_{mn}^{\text{TT}}|_{y=y^*} = 0, \quad (2.7)$$

$$\left\{ (\partial_y + 2\dot{\sigma}) B + \frac{2}{3}\kappa^3 \text{Re} \left(\frac{\partial \lambda_{\vartheta^*}}{\partial \varphi}(\varphi_{\text{cl}})\tilde{\varphi} \right) \right\} \Big|_{y=y^*} = 0, \quad (2.8)$$

where $y^* \equiv R\vartheta^* = 0, \pi R$ are the locations of the boundaries.⁵ Using the jump condition (A.2), the second conditions are seen to be equivalent to Eq.(2.6) and thus provide no new constraints.

From Eqs.(2.4) and (2.7), we can see that h_{mn}^{TT} is decomposed into the 4D massless graviton and the massive K.K. gravitons [19]. Since we are not interested in the 4D gravitational interactions, we will neglect h_{mn}^{TT} in the following.

To simplify the discussion, we will assume that the background field configuration $\varphi_{\text{cl}}(y)$ is real, and the fluctuation field $\tilde{\varphi}$ satisfies the boundary condition:

$$\tilde{\varphi}|_{y=y^*} = 0. \quad (2.9)$$

This condition is realized, for example, in the Goldberger-Wise mechanism [9] and its supersymmetric version [17] considered in the next subsection. In this case, the boundary conditions (2.8) become

$$(\partial_y + 2\dot{\sigma}) B|_{y=y^*} = 0. \quad (2.10)$$

Note that Eq.(2.6) indicates that the trace part of the metric perturbation B and the bulk scalar mode $\varphi_{\text{R}} \equiv \text{Re} \tilde{\varphi}$ describe the same physical degree of freedom. In fact, the linearized equation of motion for φ_{R} can also be derived from Eqs.(2.5) and (2.6). Using Eq.(2.6), we can eliminate $\tilde{\varphi}$ in Eq.(2.5) and obtain an equation only for B .

$$\left\{ e^{-2\sigma}\square_4 - \partial_y^2 + 2 \left(\frac{\ddot{\varphi}_{\text{cl}}}{\dot{\varphi}_{\text{cl}}} - \dot{\sigma} \right) \partial_y + 4 \left(\dot{\sigma} \frac{\ddot{\varphi}_{\text{cl}}}{\dot{\varphi}_{\text{cl}}} - \ddot{\sigma} \right) \right\} B = 0. \quad (2.11)$$

⁵Strictly speaking, the boundaries cannot be expressed by the rigid value of y in the Newton gauge [20]. However, we can express them by $y = y^*$ at the linearized order. (See Appendix B.)

We have used Eq.(A.1) again.

From Eq.(2.11), the mode equation for B is read off as

$$\left\{ -\partial_y^2 + 2 \left(\frac{\ddot{\varphi}_{\text{cl}}}{\dot{\varphi}_{\text{cl}}} - \dot{\sigma} \right) \partial_y + 4 \left(\dot{\sigma} \frac{\ddot{\varphi}_{\text{cl}}}{\dot{\varphi}_{\text{cl}}} - \ddot{\sigma} \right) \right\} f_{(p)}(y) = m_{(p)}^2 e^{-2\sigma} f_{(p)}(y), \quad (2.12)$$

where $m_{(p)}$ is the mass eigenvalue of the p -th K.K. mode. By solving Eq.(2.12) with the boundary conditions (2.8), we can decompose B into 4D K.K. modes.

$$B(x, y) = \sum_{p=0}^{\infty} f_{(p)}(y) b_{(p)}(x). \quad (2.13)$$

The boundary condition at one boundary fixes the overall normalization of the mode functions $f_{(p)}(y)$, and the condition at the other boundary determines the mass spectrum. As mentioned in Ref. [10], the boundary conditions (2.10) ensure the orthogonality of the mode functions.

2.2 Supersymmetric stabilization mechanism

In the rest of this section, we will demonstrate the mode expansion and identify the radion mode in a model of Ref. [17]. The radius stabilization mechanism in this model corresponds to a supersymmetric extension of the Goldberger-Wise mechanism [9].

The stabilization sector consists of a hypermultiplet (H, H^C) with the bulk mass m , where a chiral multiplet H (H^C) is defined as even (odd) under the orbifold parity. We will introduce the following superpotential, which provides source terms for the scalar component of H^C on the boundaries.

$$W_b \equiv \{J_0 \delta(y) - J_\pi \delta(y - \pi R)\} H, \quad (2.14)$$

where J_0 and J_π are constants. Here, we will make J_0 real by the phase redefinition of H .

Now we will find a field configuration that preserves $\mathcal{N} = 1$ supersymmetry. The Killing spinor equations are

$$\begin{aligned} \dot{\sigma} + k \left\{ 1 + \frac{\kappa^3}{2} (|h|^2 + |h^C|^2) \right\} - \frac{1}{3} \kappa^3 m (|h|^2 - |h^C|^2) &= 0, \\ \partial_y h + \left(m + \frac{3}{2} \dot{\sigma} \right) h &= 0, \\ -\partial_y h^C + \left(m - \frac{3}{2} \dot{\sigma} \right) h^C + J_0 \delta(y) - J_\pi \delta(y - \pi R) &= 0, \\ m h^C h &= 0, \end{aligned} \quad (2.15)$$

where k is a constant which becomes an AdS curvature in the limit of $J_0, J_\pi \rightarrow 0$.⁶ The scalar fields h and h^C are the scalar components of H and H^C , respectively. The first

⁶From the viewpoint of the superconformal gravity, k and m are determined by the gauge couplings for the graviphoton [16]. (See Eqs.(3.20) and (3.23).)

equation comes from the supersymmetric variation of the gravitino, the second and third ones from the hyperinos, and the last one from the graviphotino.⁷

The source terms on the boundaries lead to the boundary conditions for h^C as

$$[h^C]_0 = J_0, \quad [h^C]_\pi = J_\pi, \quad (2.16)$$

where the symbol $[\cdot\cdot\cdot]_{\vartheta^*}$ is defined by Eq.(A.3). Due to these conditions and the last equation in Eq.(2.15), only h^C can have a nonzero background. Terms involving scalar fields in the first equation of Eq.(2.15) correspond to the backreaction of the scalar configuration on the metric. We will concentrate ourselves on the case that the backreaction is sufficiently small. Then, the classical background solution can be solved as

$$\sigma(y) = -ky - \frac{l^2}{24} e^{2\gamma(y-\pi R)} + \mathcal{O}(l^4), \quad (2.17)$$

$$h_{\text{cl}}(y) = 0, \quad (2.18)$$

$$h_{\text{cl}}^C(y) = \frac{J_0}{2} e^{\gamma y} (1 + \mathcal{O}(l^2)), \quad (2.19)$$

where

$$\gamma \equiv m + \frac{3}{2}k, \quad (2.20)$$

and $l \equiv \kappa^{3/2} |J_\pi|$ is a dimensionless parameter that parametrizes the size of the backreaction.

Combining Eq.(2.19) with Eq.(2.16), we can obtain the relation

$$J_0 = J_\pi e^{-\gamma\pi R}. \quad (2.21)$$

Thus, J_π must be real for the above supersymmetric solution to exist. Eq.(2.21) fixes the radius R to the definite value determined by the ratio of J_0 and J_π . Namely, the radius is stabilized. Note that the Killing spinor equations are the first-order differential equations and their solution contains only one integration constant. It is fixed by one of the boundary conditions (2.16) and the other condition fixes the radius. However, the full equations of motion are the *second-order* differential equations and thus the most general solution contains two integration constants. In our case, the second integration constant is fixed by the minimization condition for the configuration energy (not just the stationary condition) because the preserved supersymmetry ensures the stability of the field configuration.

Since only h^C has the nonzero background configuration, we can apply the equations in the previous subsection by replacing $\varphi_{\text{cl}}(y)$ with $h_{\text{cl}}^C(y)$. If we neglect the backreaction on the metric, the mode equation (2.12) can be easily solved. In this limit, the lightest mode $b_{(0)}(x)$ is massless and its mode function is

$$f_{(0)}(y) = C_{(0)} e^{-2\sigma(y)} = C_{(0)} e^{2ky}, \quad (2.22)$$

where $C_{(0)}$ is a normalization constant. The other K.K. modes are expressed by the Bessel functions, and their mass spectrum is determined by the boundary condition (2.10) [10].

⁷The graviphotino itself is unphysical after the superconformal gauge fixing, but the corresponding Killing spinor equation must be satisfied by the supersymmetric solution.

If we take into account the backreaction, however, one can find that $b_{(0)}(x)$ obtains the following nonzero mass of order $\mathcal{O}(l^2)$ as pointed out in Ref. [10].

$$m_{(0)}^2 = \frac{l^2 k^2}{6} \left(1 - \frac{2m}{k}\right) \left(\frac{3}{2} + \frac{m}{k}\right)^2 e^{-2k\pi R} \frac{1 - e^{-2k\pi R}}{1 - e^{-(k-2m)\pi R}} + \mathcal{O}(l^4). \quad (2.23)$$

The corresponding mode function satisfies the equation

$$\partial_y f_{(0)} + 2\dot{\sigma} f_{(0)} = \mathcal{O}(l^2). \quad (2.24)$$

Thus, we can find that the mode function of $b_{(0)}(x)$ in $\varphi_R \equiv \text{Re}(h^C - h_{\text{cl}}^C)$ is suppressed by $\mathcal{O}(l)$ factor from Eq.(2.6). In fact, $b_{(0)}(x)$ enters in $\varphi_R(x, y)$ as

$$\begin{aligned} \varphi_R = & -l \frac{M_5^{\frac{3}{2}} \gamma}{2k} e^{\gamma(y-\pi R)} (e^{2ky} - 1) \left\{ 1 - e^{2(\gamma-k)(\pi R-y)} \frac{(1 - e^{-2k\pi R})(1 - e^{-(k-2m)y})}{(1 - e^{-2ky})(1 - e^{-(k-2m)\pi R})} \right\} b_{(0)}(x) \\ & + \dots, \end{aligned} \quad (2.25)$$

where the ellipsis denotes the massive K.K. modes. Note that this vanishes on the boundaries. In fact, the fluctuation field $\tilde{h}^C \equiv h^C - h_{\text{cl}}^C$ satisfies the boundary condition (2.9) due to Eq.(2.16). Thus, the mode functions satisfy the following orthonormal relation.

$$\int_0^{\pi R} dy \frac{(3M_5^3 m_{(p)})^2}{2\dot{\varphi}_{\text{cl}}^2} f_{(p)}(y) f_{(q)}(y) = \delta_{pq}. \quad (2.26)$$

The constant factors $(3M_5^3 m_{(p)})^2/2$ are determined by requiring the kinetic terms for $b_{(p)}(x)$ are canonically normalized.

2.3 Radion mode

Now, we will identify the dynamical radion mode and its appearance in the action. Remember that the scalar fluctuation mode φ_R represents the same degree of freedom as the metric perturbation B . Thus, each mode $b_{(p)}(x)$ is contained not only in the metric but also in the bulk scalar h^C . Therefore, the kinetic term for each mode comes from both the Einstein-Hilbert term and the kinetic term for h^C . From the Einstein-Hilbert term \mathcal{L}_{EH} , the following term comes out.

$$\begin{aligned} \mathcal{L}_{\text{EH}} &= \frac{M_5^3 \sqrt{g}}{2} \mathcal{R} \\ &= 3M_5^3 e^{2\sigma} \eta^{mn} \partial_m B \partial_n B + \dots \end{aligned} \quad (2.27)$$

We have performed the partial integral in the second equation.

From the scalar kinetic term $\mathcal{L}_{\text{scalar}}$, we will obtain

$$\begin{aligned} \mathcal{L}_{\text{scalar}} &= \sqrt{g} \partial^\mu \bar{h}^C \partial_\mu h^C = e^{2\sigma} \eta^{mn} \partial_m \varphi_R \partial_n \varphi_R + \dots \\ &= e^{2\sigma} \left\{ -\frac{2}{3} \kappa^3 \dot{\varphi}_{\text{cl}} \right\}^{-2} \eta^{mn} \partial_m (\partial_y B + 2\dot{\sigma} B) \partial_n (\partial_y B + 2\dot{\sigma} B) + \dots \end{aligned} \quad (2.28)$$

In the second equation, we have used Eq.(2.6).

Therefore, the kinetic term for each mode in B becomes

$$\mathcal{L}_{\text{kin}} = M_5^3 e^{2\sigma} \sum_{p,q} \left\{ 3f_{(p)}f_{(q)} + \frac{9M_5^3}{4\dot{\varphi}_{\text{cl}}^2} (\partial_y f_{(p)} + 2\dot{\sigma} f_{(p)}) (\partial_y f_{(q)} + 2\dot{\sigma} f_{(q)}) \right\} \eta^{mn} \partial_m b_{(p)} \partial_n b_{(q)} + \dots \quad (2.29)$$

The first term comes from \mathcal{L}_{EH} and the second term from $\mathcal{L}_{\text{scalar}}$. First, let us consider the lightest mode $b_{(0)}(x)$. Note that $\kappa^3 \dot{\varphi}_{\text{cl}}^2 = \mathcal{O}(l^2)$ and Eq.(2.24). Hence, the contribution from $\mathcal{L}_{\text{scalar}}$ is suppressed by $\mathcal{O}(l^2)$ comparing to that of \mathcal{L}_{EH} . For the other modes, on the other hand, the first term in Eq.(2.29) is suppressed by $\mathcal{O}(l^2)$ comparing to the second term because $\partial_y f_{(p)} + 2\dot{\sigma} f_{(p)}$ ($p \neq 0$) are not suppressed by $\mathcal{O}(l^2)$. These facts suggest that $b_{(0)}(x)$ is mainly contained in the metric while the other modes are mainly in the bulk scalar φ_{R} . After the y -integration and the partial integral, we can see that the kinetic terms for $b_{(p)}(x)$ are diagonal thanks to the mode equation (2.12) and the orthonormal relation (2.26).

Finally, we will identify the radion field. The radion field $r(x)$ is defined as the proper length along the fifth dimension. Thus, at the linear order for the fields,

$$\begin{aligned} r(x) &\equiv \frac{1}{\pi} \int_0^{\pi R} dy |g_{yy}(x, y)|^{\frac{1}{2}} = \frac{1}{\pi} \int_0^{\pi R} dy \{1 - 2B(x, y)\} \\ &= R - \frac{2}{\pi} \int_0^{\pi R} dy f_{(0)}(y) \cdot b_{(0)}(x) + \mathcal{O}(l). \end{aligned} \quad (2.30)$$

The last term $\mathcal{O}(l)$ corresponds to the massive K.K. modes $b_{(p)}$ ($p \neq 0$). Therefore, the lightest mode $b_{(0)}(x)$ can be identified with the radion fluctuation mode.

2.4 Graviphoton mode

When the model is embedded into 5D SUGRA, the theory contains the graviphoton W_μ^0 . As is well-known, the radion supermultiplet involves W_y^0 . Thus, we will consider the K.K. decomposition of W_y^0 in this subsection.

The relevant terms in the lagrangian are ⁸

$$\mathcal{L} = \sqrt{g} \left[-\frac{3}{8} M_5 F^{0\mu\nu} F_{\mu\nu}^0 + \mathcal{D}^\mu \bar{h}^C \mathcal{D}_\mu h^C \right] + \dots \quad (2.31)$$

Here, $F_{\mu\nu}^0 \equiv \partial_\mu W_\nu^0 - \partial_\nu W_\mu^0$, and $\mathcal{D}_\mu h^C$ is the covariant derivative for the graviphoton defined as

$$\mathcal{D}_\mu h^C \equiv \partial_\mu h^C + i\kappa\gamma W_\mu^0 h^C, \quad (2.32)$$

where γ is defined in Eq.(2.20). The ellipsis in Eq.(2.31) denotes irrelevant terms to the

⁸Here, we will follow the notations of Ref. [14, 18]. The graviphoton W_μ^0 is not canonically normalized there.

discussion. Then, the linearized equations of motion for W_μ^0 and φ_I are ⁹

$$e^{-2\sigma} (\Box_4 W_m^0 - \partial_m \partial_n W^{0n}) - \partial_y^2 W_m^0 - 2\dot{\sigma} \partial_y W_m^0 + (\partial_y + 2\dot{\sigma}) \partial_m W_y^0 + \frac{8}{3} \kappa^2 \{ \gamma \varphi_{\text{cl}} \partial_m \varphi_I + \kappa \gamma^2 \varphi_{\text{cl}}^2 W_m^0 \} = 0, \quad (2.33)$$

$$e^{-2\sigma} \Box_4 W_y^0 + \frac{8}{3} \kappa^3 \gamma^2 \varphi_{\text{cl}}^2 W_y^0 - e^{-2\sigma} \partial_y \partial_m W^{0m} + \frac{8}{3} \kappa^2 \gamma (\varphi_{\text{cl}} \partial_y \varphi_I - \dot{\varphi}_{\text{cl}} \varphi_I) = 0, \quad (2.34)$$

$$e^{-2\sigma} \Box_4 \varphi_I - \partial_y^2 \varphi_I - 4\dot{\sigma} \partial_y \varphi_I + \left\{ \gamma(\gamma - 4k) - \frac{\kappa^3 \gamma^2}{6} \varphi_{\text{cl}}^2 \right\} \varphi_I + e^{-2\sigma} \kappa \gamma \varphi_{\text{cl}} \partial_m W^{0m} - \kappa \gamma \{ \varphi_{\text{cl}} \partial_y W_y^0 + (2\dot{\varphi}_{\text{cl}} + 4\dot{\sigma} \varphi_{\text{cl}}) W_y^0 \} = 0, \quad (2.35)$$

where $\varphi_{\text{cl}} \equiv h_{\text{cl}}^C$, $\varphi_I \equiv \text{Im } \tilde{h}^C$ and $W^{0m} \equiv \eta^{mn} W_n^0$. Here, we have assumed that $\langle W_y^0 \rangle = 0$.

In the limit of $l \rightarrow 0$, the gauge symmetry for the graviphoton is unbroken, and all K.K. modes of W_y^0 can be gauged away except for the zero-mode. On the other hand, W_m^0 has no zero-mode because of the orbifold projection. Thus, there is no common mode in W_m^0 and W_y^0 in this limit. Therefore, from Eq.(2.33), we will obtain

$$e^{-2\sigma} (\Box_4 W_m^0 - \partial_m \partial_n W^{0n}) - \partial_y^2 W_m^0 - 2\dot{\sigma} \partial_y W_m^0 = 0. \quad (2.36)$$

In the case that the stabilization sector exists, the K.K. modes of W_y^0 cannot be gauged away because the graviphoton gauge symmetry is broken by φ_{cl} . In this case, Eq.(2.36) no longer holds. Instead, it is expected to be

$$e^{-2\sigma} (\Box_4 W_m^0 - \partial_m \partial_n W^{0n}) - \partial_y^2 W_m^0 - 2\dot{\sigma} \partial_y W_m^0 = \mathcal{O}(l^2) \quad (2.37)$$

since Eq.(2.36) recovers in the limit of $l \rightarrow 0$. Then, the following equation comes out from Eq.(2.33).

$$(\partial_y + 2\dot{\sigma}) W_y^0 = -\frac{8}{3} \kappa^2 \gamma \varphi_{\text{cl}} \varphi_I + \mathcal{O}(l^2). \quad (2.38)$$

This means that W_y^0 and φ_I describe the same degree of freedom, just like B and φ_R in Eq.(2.6). We can show that Eq.(2.38) is consistent with Eqs.(2.34) and (2.35) at the leading order for l . This implies the validity of the expectation (2.37)

Using Eq.(2.38), we can obtain the equation for only W_y^0 from Eq.(2.34).

$$\left\{ e^{-2\sigma} \Box_4 - \partial_y^2 + 2 \left(\frac{\dot{\varphi}_{\text{cl}}}{\varphi_{\text{cl}}} - \dot{\sigma} \right) \partial_y + 4\dot{\sigma} \frac{\dot{\varphi}_{\text{cl}}}{\varphi_{\text{cl}}} \right\} W_y^0 = \mathcal{O}(l^2). \quad (2.39)$$

Here, note that $\ddot{\sigma}$ and $\kappa^3 \varphi_{\text{cl}}^2$ are $\mathcal{O}(l^2)$. For the supersymmetric solution (2.17)-(2.19), this is the same equation as Eq.(2.11) at the leading order for l . This suggests that the each modes in B and W_y^0 belong to the same supermultiplet as expected. By similar discussion in Section 2.3, we can see that the lightest mode is mainly contained in W_y^0 , while the other K.K. modes are mainly in the bulk scalar φ_I . Thus, the lightest mode of W_y^0 can approximately be expressed by the gauge-invariant Wilson line,

$$w \equiv \frac{1}{\pi} \int_0^{\pi R} dy W_y^0. \quad (2.40)$$

⁹ φ_R is decoupled from W_μ^0 at the linearized order.

3 Superspace action

In this section, we will derive the $\mathcal{N} = 1$ superspace action including the radion superfield. For this purpose, we will embed the radion field $r(x)$ into the superspace action in an appropriate manner. Then, we can obtain the desired action by promoting the radion field to a chiral superfield.

In the following, we will neglect the backreaction of the bulk scalar configuration on the metric because its effects are subdominant except for the radion mass. Namely, the warp factor is assumed as $\sigma(y) = -ky$ in this section. In Section 3.5, we will show that the correct radion mass (2.23) can also be obtained without including the backreaction by calculating the radion potential. Thus, neglecting the backreaction is a practical approximation.

3.1 Radion field in superspace action

In order to express the action on the superspace, we will neglect 4D gravitational multiplet and its K.K. modes in the following. In the previous section, we have seen that the radion mode $b(x)$ ¹⁰ is contained only in the metric and its K.K. modes reside in the radius stabilizer field φ in the limit of the small backreaction. Thus, the metric can be parametrized as¹¹

$$ds^2 = e^{2F} \eta_{mn} dx^m dx^n - G^2 dy^2, \quad (3.1)$$

where

$$F = F(b(x), y), \quad G = G(b(x), y). \quad (3.2)$$

Here, we will choose the coordinate y so that

$$G(\langle b \rangle, y) = 1, \quad (3.3)$$

where $\langle b \rangle$ denotes VEV of $b(x)$. This is always possible by the redefinition of y . Then, the parameter R , which indicates the range of y , is identical to the radius of the orbifold. Since the background spacetime is the slice of AdS_5 with the curvature k ,

$$F(\langle b \rangle, y) = \sigma(y) = -ky. \quad (3.4)$$

Although $\langle b \rangle$ is fixed to some definite value by the stabilization mechanism, $b(x)$ appears in the metric as if it is a modulus field since we neglect the backreaction. Hence, the bulk spacetime remains AdS_5 with the curvature k when the background value of $b(x)$ is moved from the true value $\langle b \rangle$. In order for the metric (3.1) to have this property, F and G must satisfy the following relation.

$$G(b(x), y) = -\frac{1}{k} \partial_y F(b(x), y). \quad (3.5)$$

Note that both (1.2) and (1.3) satisfy this condition.

¹⁰In contrast to fluctuation mode $b_{(0)}(x)$ in the previous section, we will allow $b(x)$ to have a nonzero vacuum expectation value (VEV).

¹¹The off-diagonal components g_{my} can always be gauged away.

	V^0	Φ_S^0	$V^{I \neq 0}$	$\Phi_S^{I \neq 0}$	$\Phi^{2\hat{\alpha}+1}$	$\Phi^{2\hat{\alpha}+2}$
Z_2 -parity	−	+	+	−	−	+

Table 1: Orbifold parity for each superfield

Next, we will embed the radion mode into the $\mathcal{N} = 1$ superspace. We will start with the superspace action derived in our previous paper [16].¹² There, we derived 5D superspace action on the warped background directly from 5D SUGRA action. In order to incorporate the radion fluctuation mode $b(x)$ into the action, we will replace the warp factor $\sigma(y)$ and the background value of the fünfbein $\langle e_y^4 \rangle$ in Ref. [16] with the b -dependent functions F and G , *i.e.*,

$$\begin{aligned}\sigma(y) &\rightarrow F(b(x), y), \\ \langle e_y^4 \rangle &\rightarrow G(b(x), y).\end{aligned}\tag{3.6}$$

Basically, we will follow the notation of Ref. [16]. We will introduce $n_V + 1$ vector multiplets \mathcal{V}^I ($I = 0, 1, \dots, n_V$), and $n_H + 1$ hypermultiplets $\mathcal{H}^{\hat{\alpha}}$ ($\hat{\alpha} = 0, 1, \dots, n_H$). The vector multiplet $\mathcal{V}^{I=0}$ denotes the graviphoton multiplet, and the hypermultiplet $\mathcal{H}^{\hat{\alpha}=0}$ is the compensator multiplet.¹³ The remaining multiplets are physical ones. Here, we will use $\mathcal{H}^{\hat{\alpha}=1}$ as the radius stabilizer multiplet.

From the vector multiplet \mathcal{V}^I , we can construct $\mathcal{N} = 1$ vector and chiral superfields V^I and Φ^I . For simplicity, we will consider only abelian gauge groups in this paper. From the hypermultiplets $\mathcal{H}^{\hat{\alpha}}$, we can construct a pair of chiral superfields $(\Phi^{2\hat{\alpha}+1}, \Phi^{2\hat{\alpha}+2})$. The explicit form of each superfield is collected in Appendix C.1. The orbifold parity for each superfield is listed in Table. 1.

Using these superfields, the 5D superconformal invariant action on the warped geometry can be written as follows.

$$\begin{aligned}S &= \int d^5x \left(\mathcal{L}_{\text{vector}} + \mathcal{L}_{\text{hyper}} \right), \\ \mathcal{L}_{\text{vector}} &= \left[\int d^2\theta \frac{3C_{IJK}}{2} \left\{ i\Phi_S^I \mathcal{W}^J \mathcal{W}^K + \frac{1}{12} \bar{D}^2 (V^I D^\alpha \partial_y V^J - D^\alpha V^I \partial_y V^J) \mathcal{W}_\alpha^K \right\} + \text{h.c.} \right] \\ &\quad - e^{2F} \int d^4\theta G^{-2} C_{IJK} \mathcal{V}_S^I \mathcal{V}_S^J \mathcal{V}_S^K, \\ \mathcal{L}_{\text{hyper}} &= -2e^{2F} \int d^4\theta G d_\alpha{}^\beta \bar{\Phi}^\beta \left(e^{-2igV^I t_I} \right)_\gamma{}^\alpha \Phi^\gamma \\ &\quad - e^{3F} \left[\int d^2\theta \Phi^\alpha d_\alpha{}^\beta \rho_{\beta\gamma} (\partial_y - 2g\Phi_S^I t_I)^\gamma{}_\delta \Phi^\delta + \text{h.c.} \right],\end{aligned}\tag{3.7}$$

¹²This action can also be obtained from that of Ref. [15] by fixing the gravitational multiplet to its background value.

¹³In this paper, we will consider the case of one compensator multiplet, for simplicity. An extension to the multi-compensator case is straightforward.

where

$$\begin{aligned}\mathcal{V}_S^I &\equiv -\partial_y V^I - i\Phi_S^I + i\bar{\Phi}_S^I, \\ \mathcal{W}_\alpha^I &\equiv -\frac{1}{4}\bar{D}^2 D_\alpha V^I\end{aligned}\tag{3.8}$$

are the gauge invariant quantities. C_{IJK} is a real constant tensor which is completely symmetric for the indices, and $d_\alpha{}^\beta$ is a metric of the hyperscalar space and can be brought into the standard form [21]

$$d_\alpha{}^\beta = \begin{pmatrix} \mathbf{1}_2 & \\ & -\mathbf{1}_{2n_H} \end{pmatrix}.\tag{3.9}$$

The first line of $\mathcal{L}_{\text{vector}}$ corresponds to the gauge kinetic terms and the supersymmetric Chern-Simons term. Here, all the directions of the gauging are chosen to σ_3 -direction since the gauging along the other direction mixes $\Phi^{2\hat{\alpha}+1}$ and $\Phi^{2\hat{\alpha}+2}$, which have opposite Z_2 -parities. Namely, the anti-hermitian generators t_I ($I = 0, 1, \dots, n_V$) are assumed to be

$$gt_0 = -i \begin{pmatrix} g_c^0 & \\ & \mathbf{g}^0 \end{pmatrix} \otimes \sigma_3, \quad gt_I = -i \begin{pmatrix} 0 & \\ & \mathbf{g}^I \end{pmatrix} \otimes \sigma_3, \quad (I \neq 0)\tag{3.10}$$

where $\mathbf{g}^I \equiv \text{diag}(g_1^I, g_2^I, \dots, g_{n_H}^I)$ ($I = 0, 1, \dots, n_V$) are $n_H \times n_H$ matrices of the gauge couplings for the physical hypermultiplets. We will also assume that the stabilizer multiplet $\mathcal{H}^{\hat{\alpha}=1}$ is charged under only the graviphoton W_μ^0 , *i.e.*, $g_1^{I \neq 0} = 0$. Otherwise, the gauge symmetries will be broken by the background configuration of the stabilizer field.

In order to obtain the Poincaré SUGRA, we have to fix the extraneous superconformal symmetries by imposing the gauge fixing conditions, which are listed in Appendix C.3. After this gauge fixing, Eq.(3.7) reproduces the SUGRA action in Ref. [14, 18] with the radion-dependent metric (3.1) if and only if $e^{2F}G$ is independent of the 4D coordinates x^m . Considering the conditions (3.3) and (3.4), this condition can be written as

$$2F + \ln G = 2\sigma.\tag{3.11}$$

Combining Eq.(3.11) with Eq.(3.5), we can determine F and G as

$$\begin{aligned}F &= \frac{1}{2} \ln (e^{2\sigma} + \mathcal{I}(b)), \\ G &= \frac{1}{1 + e^{-2\sigma}\mathcal{I}(b)},\end{aligned}\tag{3.12}$$

where $\mathcal{I}(b)$ is some function of only $b(x)$. From Eq.(3.3), $\mathcal{I}(b)$ satisfies

$$\mathcal{I}(\langle b \rangle) = 0.\tag{3.13}$$

If we choose $\mathcal{I}(b)$ as $\mathcal{I}(b) = \tilde{b} \equiv b - \langle b \rangle$, the metric (3.1) becomes that of Ref. [8]. Changing the choice of $\mathcal{I}(b)$ just corresponds to the field redefinition of $b(x)$, and causes no physical changes. Hence, we will take $\mathcal{I}(b) = \tilde{b}$ in the following. In this case, the fluctuation mode $\tilde{b}(x)$ is not canonically normalized. The relation to the normalized field $b_{(0)}(x)$ is

$\tilde{b}(x) = 2C_{(0)}b_{(0)}(x)$, where $C_{(0)}$ is the normalization constant in Eq.(2.22).¹⁴ The resulting forms of F and G are consistent with neglecting the K.K. graviton modes. In fact, as shown in Ref. [8], no additional terms are induced up to second order in spacetime derivatives after integrating out the K.K. gravitons.

The radion field $r(x)$ is related to $\tilde{b}(x)$ as

$$\begin{aligned} r(x) &\equiv \frac{1}{\pi} \int_0^{\pi R} dy |g_{yy}|^{\frac{1}{2}} = \frac{1}{\pi} \int_0^{\pi R} \frac{dy}{1 + e^{-2\sigma}\tilde{b}(x)} \\ &= R - \frac{1}{2k\pi} \ln \left(\frac{1 + e^{2k\pi R}\tilde{b}(x)}{1 + \tilde{b}(x)} \right), \end{aligned} \quad (3.14)$$

or equivalently,

$$\tilde{b}(x) = e^{-k\pi R} \frac{\sinh k\pi (R - r(x))}{\sinh k\pi r(x)}. \quad (3.15)$$

In addition to the bulk action (3.7), we can also construct the brane actions at the boundaries, which will be discussed in Section 3.3.

3.2 Bulk action after gauge fixing

To simplify the discussion, we will consider the maximally symmetric case in the vector sector, *i.e.*,

$$C_{IJK}M^I M^J M^K = (M^{I=0})^3 - \frac{1}{2}M^{I=0} \sum_{J=1}^{n_V} (M^J)^2, \quad (3.16)$$

where M^I is a scalar component of the 5D vector multiplet \mathcal{V}^I . (See Appendix C.) Then, after the superconformal gauge fixing, the lagrangian in Eq.(3.7) becomes

$$\begin{aligned} \mathcal{L} &= \left\{ \int d^2\theta \frac{1}{4} G_c \tilde{\mathcal{W}}^X \tilde{\mathcal{W}}^X + \text{h.c.} \right\} + e^{2\sigma} \int d^4\theta G^{-2} \left(\partial_y \tilde{V}^X + i\tilde{\Phi}_S^X - i\tilde{\bar{\Phi}}_S^X \right)^2 \\ &\quad - e^{2\sigma} \int d^4\theta (\bar{\Sigma}\Sigma)^{\frac{3}{2}} \left\{ 2M_5^3 - G^{\frac{3}{2}} \left(\bar{H} e^{2\tilde{g}^X \tilde{V}^X} H + \bar{H}^C e^{-2\tilde{g}^X \tilde{V}^X} H^C \right) \right\} \\ &\quad + e^{3\sigma} \left\{ \int d^2\theta \Sigma^3 H^C \left(\frac{1}{2} \overleftrightarrow{\partial}_y + m G_c - 2i\tilde{g}^X \tilde{\Phi}_S^X \right) H + \text{h.c.} \right\} \\ &\quad + \mathcal{L}_{\text{kin}}^b, \end{aligned} \quad (3.17)$$

where the summations for the indices $X = 1, 2, \dots, n_V$ are implicit, and

$$G \equiv G(b(x), y) = \frac{1}{1 + e^{-2\sigma(y)}\tilde{b}(x)}, \quad (3.18)$$

$$G_c \equiv -2i\kappa\varphi_S^0 = G - i\kappa W_y^0. \quad (3.19)$$

Here, φ_S^0 is a scalar component of Φ_S^0 . The derivative operator $\overleftrightarrow{\partial}_y$ is defined as $A \overleftrightarrow{\partial}_y B \equiv A\partial_y B - B\partial_y A$. We have dropped quartic or higher order terms for the physical fields except

¹⁴Note that the metric (3.1) with the choice $\mathcal{I} = 2C_{(0)}b_{(0)} + \mathcal{O}(b_{(0)}^2)$ is consistent with that in the Newton gauge at the linearized order, in which we worked in the previous section.

for the radion because they are suppressed by the large Planck mass M_5 . Furthermore, we have assumed that the vacuum configuration preserves $\mathcal{N} = 1$ supersymmetry. The warp factor is determined by the Killing spinor condition from the supersymmetric variation of the gravitino, and as a result, the AdS curvature k can be written with respect to the compensator gauge coupling as ¹⁵

$$k = \frac{2M_5 g_c^0}{3}. \quad (3.20)$$

The physical superfields $\tilde{V}^X, \tilde{\Phi}_S^X$ are defined as

$$\tilde{V}^X = \sqrt{\frac{M_5}{2}} V^{I=X}, \quad \tilde{\Phi}_S^X = \sqrt{\frac{M_5}{2}} \Phi_S^{I=X}, \quad (3.21)$$

and correspondingly, the physical (dimensionful) gauge couplings $\tilde{\mathbf{g}}^X$ are defined as

$$\tilde{\mathbf{g}}^X \equiv \sqrt{\frac{2}{M_5}} \mathbf{g}^{I=X}. \quad (3.22)$$

The mass matrix \mathbf{m} is defined by

$$\mathbf{m} \equiv M_5 \mathbf{g}^0. \quad (3.23)$$

The compensator superfield Σ and the matter superfields H^v, H^{Cv} ($v = 1, 2, \dots, n_H$) are defined as

$$\begin{aligned} \Sigma &\equiv \kappa \left(\Phi^{\alpha=2} \right)^{\frac{2}{3}} = 1 - \theta^2 \mathcal{F}_\Sigma, \\ H^v &\equiv \frac{\sqrt{2} M_5^{\frac{3}{2}} G^{-\frac{3}{4}} \Phi^{\alpha=2v+2}}{\Phi^{\alpha=2}}, \\ H^{Cv} &\equiv \frac{\sqrt{2} M_5^{\frac{3}{2}} G^{-\frac{3}{4}} \Phi^{\alpha=2v+1}}{\Phi^{\alpha=2}}. \end{aligned} \quad (3.24)$$

$\Phi^{\alpha=1}$ and the fermionic component of Σ only contribute $\mathcal{O}(\kappa)$ quartic or higher order terms, which are neglected in this paper. The G -dependence of each superfield is determined so that G explicitly appears only through the form of G_c in the $d^2\theta$ -integrals. This is because only G_c can be promoted to a chiral superfield in the $d^2\theta$ -integrals. (See Eq.(3.37).)

The radion kinetic term $\mathcal{L}_{\text{kin}}^b$ comes from the cubic term for \mathcal{V}_S^0 in Eq.(3.7), and is written as

$$\mathcal{L}_{\text{kin}}^b = \frac{3}{4} e^{-2\sigma} M_5^3 G^2 \eta^{mn} \partial_m \tilde{b} \partial_n \tilde{b}. \quad (3.25)$$

In 5D SUGRA action, the same radion kinetic term comes from the Einstein-Hilbert term.

3.3 Brane actions

As well as the bulk action, we can express the brane actions localized on the boundaries in terms of the superfields [16]. If we neglect fluctuations of 4D gravitational fields, the

¹⁵Note that we have neglected the backreaction of the scalar configuration.

brane actions in Ref. [14] can be written as follows.

$$\begin{aligned}
S_{\text{brane}} &= \sum_{\vartheta^*=0,\pi} \int d^5x \, c_{\vartheta^*} \delta(y - R\vartheta^*) \mathcal{L}_{\text{brane}}^{(\vartheta^*)}, \\
\mathcal{L}_{\text{brane}}^{(\vartheta^*)} &= \left\{ \int d^2\theta \, f_{AB}^{(\vartheta^*)}(S) \mathcal{W}^A \mathcal{W}^B + \text{h.c.} \right\} - e^{2F} \int d^4\theta \, \bar{\Sigma} \Sigma \exp \left\{ -K^{(\vartheta^*)}(S, \bar{S}, V) \right\} \\
&\quad + e^{3F} \left\{ \int d^2\theta \, \Sigma^3 P^{(\vartheta^*)}(S) + \text{h.c.} \right\},
\end{aligned} \tag{3.26}$$

where c_{ϑ^*} are some dimensionless constants which are assumed as small numbers. $f_{AB}^{(\vartheta^*)}$, $K^{(\vartheta^*)}$ and $P^{(\vartheta^*)}$ are the (brane-localized) gauge kinetic functions, Kähler potentials, and superpotentials, respectively. The superfields V^A and S^a are the vector and chiral superfields constructed from 4D superconformal multiplets. The explicit forms of them are collected in Appendix C.2. The indices A and a run over not only the brane-localized multiplets but also induced ones on the boundaries from the bulk multiplets. For example, as chiral superfields on the boundary $y = y^*$,

$$S^v \equiv M_5^{-\frac{1}{2}} G^{\frac{3}{4}} H^v \Big|_{y=y^*} \tag{3.27}$$

can appear in the brane action (3.26).¹⁶ The G -dependence is fixed by the requirement that the Weyl weights of S^v must be zero [22]. The warp factors in $\mathcal{L}_{\text{brane}}^{(\vartheta^*)}$ come from the induced metric on the boundaries.

3.4 Promotion to the radion superfield

The radion fluctuation field $\tilde{b}(x)$ in G and G_c should be promoted to a superfield because the unbroken $\mathcal{N} = 1$ supersymmetry exists. Note that there is still an ambiguity of the field redefinition of the radion field before promoting it to the superfield. As shown in Ref. [8], the proper length $r(x)$ is suitable for the superfield description.

Using Eq.(3.15), we can rewrite $\mathcal{L}_{\text{kin}}^b$ in terms of $r(x)$ as follows.

$$\mathcal{L}_{\text{kin}}^r = \frac{3}{16} M_5^3 (k\pi)^2 (1 - e^{-2k\pi R})^2 \frac{e^{-2\sigma} G^2(r)}{\sinh^4 k\pi r} \eta^{mn} \partial_m r \partial_n r, \tag{3.28}$$

where $G(r) \equiv G(\tilde{b}(r))$.

In the limit of the small backreaction (*i.e.*, $l \rightarrow 0$), we have seen in Section 2.4 that W_y^0 has only the zero-mode, which is described by the Wilson line w defined in Eq.(2.40). By solving the equation of motion¹⁷, we can find that w and \tilde{b} are contained in W_μ^0 as [8]

$$W_y^0 = \frac{2k\pi e^{-2\sigma}}{e^{2k\pi R} - 1} \cdot \frac{(1 + \tilde{b})(1 + e^{2k\pi R} \tilde{b})}{(1 + e^{-2\sigma} \tilde{b})^2} \cdot w, \tag{3.29}$$

$$W_m^0 = \int_0^y dy' \, \partial_m \left\{ \frac{2k\pi e^{-2\sigma(y')}}{e^{2k\pi R} - 1} \cdot \frac{(1 + \tilde{b})(1 + e^{2k\pi R} \tilde{b})}{(1 + e^{-2\sigma(y')} \tilde{b})^2} \right\} w. \tag{3.30}$$

¹⁶ H^{Cv} are odd under the orbifold parity and vanish on the boundaries.

¹⁷Since w is not contained in the bulk scalar φ in the limit of $l \rightarrow 0$, we can drop the bulk scalar terms in the equation of motion.

Note that W_m^0 does not contain w at the linearized order for the fluctuation fields, which is consistent with the analysis in Section 2.4.

Substituting this expression into the kinetic term for W_y^0 in Eq.(2.31) and rewriting $b(x)$ by $r(x)$, we can obtain

$$\mathcal{L}_{\text{kin}}^w = \frac{3}{16} M_5 (k\pi)^2 (1 - e^{-2k\pi R})^2 \frac{e^{-2\sigma} G^2(r)}{\sinh^4 k\pi r} \eta^{mn} \partial_m w \partial_n w. \quad (3.31)$$

Thus, if we define a 4D complex scalar τ as

$$\tau \equiv r + i\kappa w, \quad (3.32)$$

the above kinetic terms $\mathcal{L}_{\text{kin}}^r$ and $\mathcal{L}_{\text{kin}}^w$ can be collected in the Kähler form. In order to see this explicitly, let us derive the kinetic term for τ in 4D effective theory by performing the y -integral.

$$\mathcal{L}_{\text{kin}}^{(4)} = \frac{3M_5^3 k\pi^2}{8} \frac{1 - e^{-2k\pi R}}{\sinh^2 k\pi \text{Re } \tau} \eta^{mn} \partial_m \bar{\tau} \partial_n \tau. \quad (3.33)$$

This is certainly the Kähler form with the Kähler potential

$$K_{\text{rad}}^{\text{eff}}(\tau, \bar{\tau}) = -3M_{\text{P}}^2 \ln(1 - e^{-k\pi(\tau + \bar{\tau})}). \quad (3.34)$$

Here, $M_{\text{P}} \equiv (M_5^3(1 - e^{-2k\pi R})/(2k))^{1/2}$ is the 4D effective Planck mass. This suggests that the appropriate definition of the radion field for the promotion to a chiral superfield is the proper length $r(x)$,¹⁸ and all $r(x)$ in the chiral superspace should be associated with $w(x)$ in the form of τ . Namely, G_c defined in Eq.(3.19) should be understood as

$$G_c(r, w) = G(\tau) = \left\{ 1 + e^{-2\sigma(y)} e^{-k\pi R} \frac{\sinh k\pi(R - \tau)}{\sinh k\pi\tau} \right\}^{-1}. \quad (3.35)$$

Here, we have used the relation (3.15). Then, from Eq.(3.19), G and W_y^0 are identified as

$$G \equiv \text{Re } G(\tau), \quad W_y^0 \equiv -M_5 \text{Im } G(\tau). \quad (3.36)$$

These are identical to the expressions in Eqs.(3.18) and (3.29) up to the linear order for w , but deviate from them beyond the linear order for w . This is not a problem because Eqs.(3.18) and (3.29) are solutions of the equations of motion *up to the linear order* for w . In fact, we have implicitly assumed that the metric is independent of w , but this is only valid at the linearized level. Beyond the linearized order, the Einstein equation involves W_y^0 and the metric also has the w -dependence. Thus, Eq.(3.36) provides the modified expressions of G and W_y^0 which are valid at all orders for r and w .

The appearance of w in Eq.(3.30) induces nonvanishing contributions to the action from F_{mn}^0 . In Refs. [8, 23], such contributions are ignored because they are of higher order in the derivative expansion. In our superspace formalism, on the other hand, W_m^0 is dropped from the beginning because it cannot be incorporated into the superspace. Thus, no such higher derivative terms appear in our superspace action.

¹⁸For other definitions of the radion field, for example $b(x)$, the kinetic term cannot be written as the Kähler form.

Now we will promote the complex scalar field $\tau(x)$ to a chiral superfield $T(x, \theta)$. Namely, G_c and G in Eq.(3.17) are promoted as

$$\begin{aligned} G_c &\rightarrow G(T) = \left\{ 1 + e^{2ky} e^{-k\pi R} \frac{\sinh k\pi(R-T)}{\sinh k\pi T} \right\}^{-1}, \\ G &\rightarrow G_R \equiv \text{Re } G(T). \end{aligned} \quad (3.37)$$

As a result, the bulk lagrangian becomes

$$\begin{aligned} \mathcal{L} = & \left\{ \int d^2\theta \frac{1}{4} G(T) \tilde{\mathcal{W}}^X \tilde{\mathcal{W}}^X + \text{h.c.} \right\} + e^{2\sigma} \int d^4\theta G_R^{-2} \left(\partial_y \tilde{V}^X + i\tilde{\Phi}_S^X - i\tilde{\bar{\Phi}}_S^X \right)^2 \\ & + e^{2\sigma} \int d^4\theta G_R^{\frac{3}{2}} \left(\bar{H} e^{2\tilde{\mathbf{g}}^X \tilde{V}^X} H + \bar{H}^C e^{-2\tilde{\mathbf{g}}^X \tilde{V}^X} H^C \right) \\ & + e^{3\sigma} \left\{ \int d^2\theta H^C \left(\frac{1}{2} \overleftrightarrow{\partial}_y + \mathbf{m} G(T) - 2i\tilde{\mathbf{g}}^X \tilde{\Phi}_S^X \right) H + \text{h.c.} \right\} \\ & + e^{2\sigma} \int d^4\theta K_{\text{rad}}(T, \bar{T}), \end{aligned} \quad (3.38)$$

where the radion Kähler potential $K_{\text{rad}}(T, \bar{T})$ is

$$K_{\text{rad}}(T, \bar{T}) = -3M_5^3 \ln G_R. \quad (3.39)$$

Since we assume the supersymmetric stabilization mechanism, \mathcal{F}_Σ does not have a non-zero VEV. Hence, from now on, we will drop the Σ -dependence. The 4D radion Kähler potential $K_{\text{rad}}^{(4)}(T, \bar{T})$ in the effective theory is obtained by the y -integration.

$$K_{\text{rad}}^{(4)}(T, \bar{T}) \equiv \int_0^{\pi R} dy e^{2\sigma} K_{\text{rad}}(T, \bar{T}). \quad (3.40)$$

We will not show its explicit expression here because it is lengthy and complicated. Although $K_{\text{rad}}^{(4)}(T, \bar{T})$ has a different form from $K_{\text{rad}}^{\text{eff}}(T, \bar{T})$ in Eq.(3.34), the corresponding Kähler metric is identical to that of $K_{\text{rad}}^{\text{eff}}(T, \bar{T})$ up to the linear order for $\text{Im } T$.

$$\frac{\partial^2}{\partial T \partial \bar{T}} K_{\text{rad}}^{(4)}(T, \bar{T}) = \frac{\partial^2}{\partial T \partial \bar{T}} K_{\text{rad}}^{\text{eff}}(T, \bar{T}) + \mathcal{O}((\text{Im } T)^2). \quad (3.41)$$

Since the zero-modes of the gauge superfields \tilde{V}^X have constant mode functions, the radion couplings to the superfield strengths of the zero-modes $\tilde{\mathcal{W}}_{(0)}^X$ in 4D effective lagrangian becomes simple.

$$\mathcal{L}_{\text{gauge}}^{(4)} = \int_0^{\pi R} dy \left\{ \int d^2\theta \frac{1}{4} G(T) \tilde{\mathcal{W}}^X \tilde{\mathcal{W}}^X + \text{h.c.} \right\} = \left\{ \int d^2\theta \frac{\pi}{4} T \tilde{\mathcal{W}}_{(0)}^X \tilde{\mathcal{W}}_{(0)}^X + \text{h.c.} \right\} + \cdots, \quad (3.42)$$

where the ellipsis denotes terms involving the massive K.K. modes. This coincides with the radion couplings of Ref. [24]. On the other hand, the couplings to the other K.K. modes have complicated forms due to their nontrivial mode functions.

Next, we will derive the boundary superspace lagrangians. The radion field $r(x)$ contained in F in Eq.(3.26) should be promoted to the superfield T . The resulting boundary lagrangians are

$$\begin{aligned} \mathcal{L}_{\text{brane}}^{(\vartheta^*)} = & \left\{ \int d^2\theta f_{AB}^{(\vartheta^*)}(S) \mathcal{W}^A \mathcal{W}^B + \text{h.c.} \right\} - e^{2\sigma(y^*)} \int d^4\theta \frac{1}{\text{Re } G_{\vartheta^*}(T)} \exp \{ -K^{(\vartheta^*)}(S, \bar{S}, V) \} \\ & + e^{3\sigma(y^*)} \left\{ \int d^2\theta G_{\vartheta^*}^{-\frac{3}{2}}(T) P^{(\vartheta^*)}(S) + \text{h.c.} \right\}, \end{aligned} \quad (3.43)$$

where $\vartheta^* = 0, \pi$, and

$$\begin{aligned} G_0(T) &\equiv G(T)|_{y=0} = \frac{1 - e^{-2k\pi T}}{1 - e^{-2k\pi R}}, \\ G_\pi(T) &\equiv G(T)|_{y=\pi R} = \frac{e^{2k\pi T} - 1}{e^{2k\pi R} - 1}. \end{aligned} \quad (3.44)$$

In the $d^2\theta$ -integral, this is the only way of promoting $G(r)$ because of the holomorphicity. In the $d^4\theta$ -integral, on the other hand, it seems that there is an ambiguity in the promotion. In fact, $G(r)$ might be promoted to $G(\text{Re } T)$ instead of $\text{Re } G(T)$. This ambiguity is removed by the requirement that the superspace action reproduces the brane-localized radion kinetic terms which originate from the 4D Einstein-Hilbert terms for the induced metric. Only the promotion applied in Eq.(3.43) reproduces those terms. After the promotion of $r(x)$, note that the definition of S^v in Eq.(3.27) should be modified as

$$S^v \equiv M_5^{-\frac{1}{2}} G^{\frac{3}{4}}(T) H^v \Big|_{y=y^*} \quad (3.45)$$

because of the holomorphicity.

Here, note that Eq.(3.43) includes the Wilson line w through the T -dependence. This mode is originally contained in Φ_S^0 in Eq.(3.7). On the other hand, the superconformal multiplets corresponding to Φ_S^I in our notation can be absent in the brane actions in Ref. [14]. (See Eq.(5.1) in Ref. [14].) The way of introducing Φ_S^I in the brane action is not mentioned in Ref. [14] just because it is a hard task to find how they appear in the 4D action formulae in a 5D superconformal-invariant way due to their nontrivial transformation properties. Namely, the possibility of the appearance of Φ_S^I in the brane action is not excluded yet. The appearance of w in Eq.(3.43) suggests the appearance of Φ_S^0 in the boundary action in Ref. [14]. At least, from our superspace approach, it is inevitable for w to appear in the boundary actions due to the existence of the radion mode in the induced metric.

Finally, we will comment on the explicit forms of the components in each superfield listed in Appendix C after the radion r is promoted to the superfield T . After the promotion, the expressions in Appendix C receive some modifications. They will newly obtain the dependence of w and the zero-mode of the gravitino ψ_y . However, such explicit expressions are irrelevant to the discussions once the superfield description is completed.

3.5 Radion mass

In Section 2.2, we have calculated the non-zero radion mass by including the backreaction of the scalar configuration on the metric. Here, we will show that we can also obtain the correct radion mass from the radion potential without including the backreaction.

First, we will redefine the superfields as follows.

$$\begin{aligned} H &\rightarrow G^{-\frac{m}{2k}}(T)H, \\ H^C &\rightarrow G^{\frac{m}{2k}}(T)H^C. \end{aligned} \quad (3.46)$$

Then, the superspace lagrangian of the model discussed in Section 2.2 can be written as

$$\begin{aligned} \mathcal{L} = & e^{2\sigma} \int d^4\theta G_R^{\frac{3}{2}} \left(|G^{-\frac{m}{2k}}(T)H|^2 + |G^{\frac{m}{2k}}(T)H^C|^2 \right) \\ & + e^{3\sigma} \left\{ \int d^2\theta H^C \left(\frac{1}{2} \overleftrightarrow{\partial}_y + m \right) H + \text{h.c.} \right\} + e^{2\sigma} \int d^4\theta K_{\text{rad}}(T, \bar{T}) \\ & + \delta(y) \mathcal{L}^{(0)} + \delta(y - \pi R) \mathcal{L}^{(\pi)}, \end{aligned} \quad (3.47)$$

where K_{rad} is defined in Eq.(3.39), and the boundary lagrangians are

$$\mathcal{L}^{(\vartheta^*)} = e^{3\sigma} \left\{ \int d^2\theta G^{-\left(\frac{3}{4} + \frac{m}{2k}\right)}(T) e^{i\vartheta^*} J_{\vartheta^*} H + \text{h.c.} \right\} \Big|_{y=y^*}. \quad (3.48)$$

The brane-localized Kähler potentials are not introduced.

Note that the T -dependence disappears from the $d^2\theta$ -integration in the bulk action by the redefinition (3.46). This makes it easier to calculate the radion potential. A naive way of deriving the low-energy effective theory is simply dropping the massive K.K. modes from the beginning and performing the y -integral. However, the effective theory obtained by such ‘zero-mode truncation’ may receive some corrections in the process of integrating out the massive K.K. modes. The massive modes are integrated out by using their equations of motion. If a linear term for a massive K.K. mode includes a light mode like the radion T , the procedure of integrating out such a mode induces a correction to the effective potential [25]. In Eq.(3.47), however, note that such T -dependent linear terms for the massive K.K. modes of H or H^C appears only in the boundary terms except for the Kähler terms.¹⁹ Since H^C vanishes on the boundaries, the above-mentioned corrections to the radion potential are not induced in our case.²⁰ Therefore, the naive zero-mode truncation can be used to calculate the radion potential.

By dropping the massive K.K. modes, the superfields H and H^C become

$$\begin{aligned} H(x, y, \theta) &= C_{(0)}^H e^{\left(\frac{3}{2}k - m\right)y} \cdot H_{(0)}(x, \theta), \\ H^C(x, y, \theta) &= h_{\text{cl}}^C(y) = \frac{J_0}{2} e^{\gamma y}, \end{aligned} \quad (3.49)$$

where $C_{(0)}^H$ is a normalization factor of the mode function, and $\gamma \equiv \frac{3}{2}k + m$. As mentioned in Section 2.3, the zero-mode of H^C is suppressed by an $\mathcal{O}(l)$ factor, and can be neglected.

Then, the effective 4D lagrangian is obtained by the y -integral as

$$\mathcal{L}^{(4)} = \int d^4\theta \mathcal{K}^{(4)} + \left\{ \int d^2\theta P^{(4)} + \text{h.c.} \right\}, \quad (3.50)$$

¹⁹The T -dependence in the kinetic terms of the massive modes is harmless because its effect is suppressed in low energies.

²⁰The procedure of integrating out $H_{(p)}$ ($p \neq 0$) does not induce extra T -dependence.

where

$$\begin{aligned}
\mathcal{K}^{(4)} &= K_{\text{rad}}^{(4)}(T, \bar{T}) + \int_0^{\pi R} dy e^{(k+2m)y} G_{\text{R}}^{\frac{3}{2}} \left| \frac{J_0}{2} G_{\frac{2k}{2}}^m(T) \right|^2, \\
&\quad + \int_0^{\pi R} dy e^{(k-2m)y} G_{\text{R}}^{\frac{3}{2}} \left| G^{-\frac{m}{2k}}(T) C_{(0)}^H H_{(0)} \right|^2 \\
P^{(4)} &= \frac{1}{2} G_0^{-\frac{\gamma}{2k}}(T) (J_0 - J_\pi e^{-\gamma\pi T}) C_{(0)}^H H_{(0)}.
\end{aligned} \tag{3.51}$$

Here, $K_{\text{rad}}^{(4)}$ is defined in Eq.(3.40). The superpotential $P^{(4)}$ originates only from the boundary terms (3.48). Note that the factor $\frac{1}{2}$ in $P^{(4)}$ comes from the y -integral of the δ -functions since the integral interval is taken as $[0, \pi R]$.

The scalar potential V_{scalar} is calculated from this as

$$V_{\text{scalar}} = (g_K^{-1})^{a\bar{b}} P_a^{(4)} \bar{P}_{\bar{b}}^{(4)}, \tag{3.52}$$

where $a, b = \tau, h_{(0)}$ ($h_{(0)}$ is a scalar component of $H_{(0)}$), $P_a^{(4)} \equiv \partial P^{(4)} / \partial a, \dots$ and $(g_K)_{a\bar{b}} \equiv \mathcal{K}_{a\bar{b}}^{(4)}$ is the Kähler metric. We have dropped contributions from \mathcal{F}_Σ since they are irrelevant to the following discussion. The minimization conditions of this potential lead to the following vacuum.

$$\begin{aligned}
\langle h_{(0)} \rangle &= 0, \\
J_0 - J_\pi e^{-\gamma\pi\langle\tau\rangle} &= 0.
\end{aligned} \tag{3.53}$$

Thus, the radius $\langle r \rangle = \text{Re} \langle \tau \rangle$ is certainly stabilized to a finite value. The second equation reduces to Eq.(2.21) since $\langle r \rangle = R$.

For the calculation of the radion mass, we can restrict the potential to the section of $h_{(0)} = 0$. Then, the radion potential V_{rad} reduces to the following simple form.

$$\begin{aligned}
V_{\text{rad}}(\tau, \bar{\tau}) &= \left(\mathcal{K}_{H_{(0)} \bar{H}_{(0)}}^{(4)} \right)^{-1} \left| P_{H_{(0)}}^{(4)} \right|^2 \Big|_{h_{(0)}=0} \\
&= \frac{\left| \frac{1}{2} G_0^{-\frac{\gamma}{2k}}(\tau) \right|^2}{\int_0^{\pi R} dy e^{(k-2m)y} G_{\text{R}}^{\frac{3}{2}} \left| G^{-\frac{m}{2k}}(\tau) \right|^2} \cdot \left| J_0 - J_\pi e^{-\gamma\pi\tau} \right|^2 + \mathcal{O}(l^4),
\end{aligned} \tag{3.54}$$

where $l \equiv \kappa^{3/2} |J_\pi|$.

Considering canonical normalization of the radion kinetic term, we can calculate the radion mass as

$$\begin{aligned}
m_{\text{rad}}^2 &= \left(\mathcal{K}_{T\bar{T}}^{(4)} \right)^{-1} \frac{\partial^2 V_{\text{rad}}}{\partial \tau \partial \bar{\tau}} \Big|_{\tau=R} \\
&= \frac{l^2 k^2}{6} \left(1 - \frac{2m}{k} \right) \left(\frac{3}{2} + \frac{m}{k} \right)^2 e^{-2k\pi R} \frac{1 - e^{-2k\pi R}}{1 - e^{-(k-2m)\pi R}} + \mathcal{O}(l^4).
\end{aligned} \tag{3.55}$$

This radion mass is exactly identical to Eq.(2.23) that is obtained by solving the mode equation. This supports the validity of the T -dependence of the action obtained in the

previous subsection. In Ref. [17], the radion mass is calculated by using the superspace action of Ref. [13], which is based on the naive ansatz (1.2). Their result is similar to ours if $kR \gtrsim 1$.²¹ (For example, $kR \simeq 12$ in the original Randall-Sundrum model.)

In the above derivation of the radion mass, the backreaction on the warp factor contributes only to a higher order correction of $\mathcal{O}(l^4)$, in contrast to the derivation in Section 2.2. Now, the leading $\mathcal{O}(l^2)$ contribution comes from the scalar configuration $h_{\text{cl}}^C(y)$, not from the backreaction. Therefore, we can obtain the correct radion mass without including the backreaction term in the warp factor.

4 Summary

We have derived 5D superspace action including the *dynamical* radion superfield, and clarified its couplings to the bulk and the boundary matter superfields. The resulting superspace lagrangian is provided by Eqs.(3.38) and (3.43) with Eq.(3.37). Our result is obtained in a systematic way based on the superconformal formulation of 5D SUGRA in Ref. [14, 18].

The T -dependence of our action is different from that of Ref. [13], which is based on the naive ansatz (1.2). As we mentioned at the end of Section 3.5, the correct order of the radion mass can also be obtained by using the action of Ref. [13] if $kR \gtrsim 1$, although it slightly deviates from the correct value (2.23) by a factor $(1 - e^{-2k\pi R})$. This is because the radion potential reflects only infrared behavior of the radion field $r(x)$, and the radion field defined in Ref. [13] has a common infrared behavior with ours. (VEV of either radion field corresponds to the radius of the orbifold.) However, when the radion field is treated as a dynamical degree of freedom, the difference of its definition becomes relevant. For example, the radion couplings to the K.K. modes of the matter fields are quite different from those of Ref. [13]. In that case, calculations should be performed by using our result.

Note that $K_{\text{rad}}^{(4)}(T, \bar{T})$ defined in Eq.(3.40) is different from $K_{\text{rad}}^{\text{eff}}(T, \bar{T})$ in Eq.(3.34), which is derived in Refs.[8, 23]. This difference cannot be removed by the redefinition of the superfields. The most general form of $G(T)$ in Eq.(3.37) after the redefinition of T is written in the form

$$G(T) = \frac{1}{1 + e^{2ky} f(T)}, \quad (4.1)$$

where $f(T)$ is a holomorphic function of T and satisfies $\langle f(T) \rangle = 0$. Thus we can calculate the most general form of $K_{\text{rad}}^{(4)}(T, \bar{T})$ by substituting the above expression into Eqs.(3.39) and (3.40) and performing the y -integration. Then, we can easily see that $K_{\text{rad}}^{(4)}(T, \bar{T})$ cannot be reduced to $K_{\text{rad}}^{\text{eff}}(T, \bar{T})$ in Eq.(3.34) except for the flat case, no matter how we choose the function $f(T)$. Only in the flat case, the former is reduced to the latter without any redefinition of T . We will comment on this case at the end of this section.

In Ref. [23], it is concluded that the Kähler potential is a function of only $\text{Re } \tau$ from the invariance of the bosonic part of the action under a constant shift of W_y^0 . However, we should emphasize that the constant shift of W_y^0 does not correspond to a constant

²¹The radion mass calculated in Ref. [17] is factor two larger than our result besides an extra factor $(1 - e^{-2k\pi R})$, but we think that this factor two is just their calculation error.

shift of its zero-mode w . (See Eq.(3.29), for example.) Note that w in Eq.(3.32) must be a *fluctuation mode*. If it has a nonzero VEV, G and W_y^0 defined by Eq.(3.36) deviate from Eqs.(3.18) and (3.29) even at the leading order for the fluctuation modes. Thus, the constant shift of w is not allowed. In the pure supergravity case discussed in Ref. [8], the bosonic part of the action is certainly invariant under the constant shift of W_y^0 . However, such constant shift affects only VEV of W_y^0 and not the fluctuation mode w . Recall that we have assumed $\langle W_y^0 \rangle = 0$ in Eq.(2.40). If we admit a nonzero $\langle W_y^0 \rangle$, Eq.(2.40) should be written as

$$w \equiv \frac{1}{\pi} \int_0^{\pi R} dy (W_y^0 - \langle W_y^0 \rangle). \quad (4.2)$$

Therefore, we conclude that the ‘mode’ which should not appear in $K_{\text{rad}}^{(4)}(\tau, \bar{\tau})$ is not $w = M_5 \text{Im } \tau$, but $W_y^0 = -M_5 \text{Im } G(\tau)$.

In the last section of Ref. [8], another derivation of the radion Kähler potential $K_{\text{rad}}^{\text{eff}}(T, \bar{T})$ is presented, which is based on the assumption that 4D effective theory is described by 4D Einstein supergravity. However, when the radion is dynamical, the gravity deviates from the ordinary 4D Einstein gravity since the radion behaves like a Brans-Dicke scalar [5, 6].²² Therefore, the discussion there may not be applicable to the derivation of the radion Kähler potential. Thus, the deviation of our result from that of Ref. [8] does not lead to an immediate contradiction.

In this paper, we have assumed the supersymmetric radius stabilization, and dropped the dependence of the compensator superfield. Actually, the F -terms of the compensator and the radion superfields are closely related to each other, and the promotion of the radion to the superfield involves a modification of \mathcal{F}_Σ . This fact becomes relevant when we consider the Scherk-Schwarz breaking of the supersymmetry [26]. We will discuss this issue in the subsequent paper.

Finally, we will comment on the flat limit (*i.e.*, $k \rightarrow 0$). In this limit, $G(T)$ becomes independent of y and reduces to a simple form,

$$G(T) = \frac{T}{R}. \quad (4.3)$$

Then, the T -dependence of the action becomes greatly simplified. Furthermore, from Eqs.(3.39) and (3.40), the 4D radion Kähler potential becomes the following no-scale form up to a constant.

$$K_{\text{rad}}^{(4)}(T, \bar{T}) = -3M_{\text{P}}^2 \ln (T + \bar{T}), \quad (4.4)$$

where $M_{\text{P}} = (\pi R M_5^3)^{1/2}$ is the 4D Planck mass.

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²²Of course, the Einstein gravity will recover below the radion mass scale.

A Equations of motion for the background

The background (2.2) satisfies the following equations.

$$\begin{aligned}\dot{\sigma}^2 &= \frac{\kappa^3}{6} \{ |\dot{\varphi}_{\text{cl}}|^2 - V(\varphi_{\text{cl}}) \}, \\ \ddot{\sigma} &= -\frac{\kappa^3}{3} \left\{ 2 |\dot{\varphi}_{\text{cl}}|^2 + \sum_{\vartheta^*=0,\pi} \lambda_{\vartheta^*}(\varphi_{\text{cl}}) \delta(y - R\vartheta^*) \right\}, \\ \ddot{\varphi}_{\text{cl}} + 4\dot{\sigma}\dot{\varphi}_{\text{cl}} &= \frac{\partial V}{\partial \varphi}(\varphi_{\text{cl}}) + \sum_{\vartheta^*=0,\pi} \frac{\partial \lambda_{\vartheta^*}}{\partial \varphi}(\varphi_{\text{cl}}) \cdot \delta(y - R\vartheta^*),\end{aligned}\tag{A.1}$$

where the dot denotes the derivative with respect to y , and $\vartheta^* = 0, \pi$ are the brane locations in the dimensionless coordinate ϑ . The first two equations come from the Einstein equation and the last one is the equation of motion for φ . By integrating the above equations over infinitesimal regions including the boundaries $y = y^*(= R\vartheta^*)$, we can obtain the following jump conditions.

$$\begin{aligned}[\dot{\sigma}]_{\vartheta^*} &= -\frac{\kappa^3}{3} \lambda_{\vartheta^*}(\varphi_{\text{cl}}) \Big|_{y=y^*}, \\ [\dot{\varphi}_{\text{cl}}]_{\vartheta^*} &= \frac{\partial \lambda_{\vartheta^*}}{\partial \varphi}(\varphi_{\text{cl}}) \Big|_{y=y^*},\end{aligned}\tag{A.2}$$

where $[\cdots]_{\vartheta^*}$ is defined as

$$[\alpha(y)]_{\vartheta^*} \equiv \alpha(y^* + 0) - \alpha(y^* - 0).\tag{A.3}$$

B Boundaries in the Newton gauge

In this appendix, we will show that the boundaries can be expressed by the rigid values of the coordinate y within the Newton gauge *at the linearized order*.

To describe the boundary conditions, the Gaussian normal (GN) coordinates are useful [20] because the boundary is expressed by the rigid value of y . In this gauge, however, two coordinate patches are necessary for covering the whole spacetime. The first patch contains one boundary and the second one contains the other boundary whose locations are expressed by $y = y^+(x)$ and $y = y^-(x)$ in the Newton gauge, respectively. We can move from the Newton gauge to the GN gauge by the following transformation.

$$x_{\text{GN}}^\mu = x_{\text{Newton}}^\mu + \xi^\mu,\tag{B.1}$$

where the transformation parameters $\xi^\mu(x, y)$ are

$$\begin{aligned}\xi_y(x, y) &= 2 \int_{y^\pm}^y dy' B(x, y') + \xi_y^\pm(x), \\ \xi_m(x, y) &= -2 \int_{y^\pm}^y dy' e^{-2\sigma(y')} \int_{y^\pm}^{y'} dy'' \partial_m B(x, y'') \\ &\quad - \{ \partial_m \xi_y^\pm - 2\partial_m y^\pm B(x, y^\pm) \} \int_{y^\pm}^y dy' e^{-2\sigma(y')} + \xi_m^\pm(x).\end{aligned}\tag{B.2}$$

Here, $B = \frac{1}{4}h_{yy}$ is the fluctuation of the metric g_{yy} around the background in the Newton gauge (See Eq.(2.3).), and $y^\pm(x)$ and $\xi_\mu^\pm(x)$ are functions of only x^m . From Eq.(B.1) and the first equation of Eq.(B.2), we can see that

$$y^\pm(x) + \xi_y^\pm(x) = \text{constant} \quad (\text{B.3})$$

because the boundaries are expressed by $y = \text{constant}$ in the GN coordinates.

By using the above transformation, we can obtain the following boundary conditions in the Newton gauge [20, 19].

$$\begin{aligned} \partial_y h_{mn}^{\text{TT}} + 2e^{-2\sigma} \partial_m \partial_n \xi_y^\pm &= 0, \\ \partial_y B + 2\dot{\sigma} B + \ddot{\sigma} \xi_y^\pm &= 0 \end{aligned} \quad (\text{B.4})$$

at $y = y^\pm$. For simplicity, we have assumed that the fluctuation $\tilde{\varphi}$ vanishes at the boundaries. Taking the trace of the first condition, we can find that $\square_4 \xi_y^\pm = 0$.

By solving the linearized Einstein equation, the scalar perturbation of the metric B is expanded into the K.K. modes.

$$B(x, y) = \sum_p f_{(p)}(y) b_{(p)}(x), \quad (\text{B.5})$$

where each mode satisfies

$$\square_4 b_{(p)} = m_{(p)}^2 b_{(p)} + \mathcal{O}(b^2). \quad (\text{B.6})$$

Here, $\square_4 \equiv \eta^{mn} \partial_m \partial_n$, and $m_{(p)}$ is the mass eigenvalue of the p -th K.K. mode. Plugging Eq.(B.5) into the second condition in Eq.(B.4), we can obtain

$$\sum_k (\partial_y f_{(p)} + 2\dot{\sigma} f_{(p)}) b_{(p)} = -\ddot{\sigma} \xi_y^\pm. \quad (\text{B.7})$$

Operating \square_4 on the both sides,

$$\sum_{p'} (\partial_y f_{(p')} + 2\dot{\sigma} f_{(p')}) m_{(p')}^2 b_{(p')} = 0 \quad (\text{B.8})$$

at the linearized level. Here, p' does not include the massless modes, *i.e.*, $m_{(p')}^2 \neq 0$. We have used $\square_4 \xi_y^\pm = 0$ and Eq.(B.6).

Since $b_{(p)}(x)$ are independent fields, Eq.(B.8) means that

$$(\partial_y f_{(p')} + 2\dot{\sigma} f_{(p')})_{y=y^\pm} = 0. \quad (\text{B.9})$$

Therefore, Eq.(B.7) becomes

$$\sum_{m_{(p)}^2=0} (\partial_y f_{(p)} + 2\dot{\sigma} f_{(p)}) b_{(p)} = -\ddot{\sigma} \xi_y^\pm. \quad (\text{B.10})$$

On the other hand, there is no physical massless mode in B due to the stabilization mechanism.²³ For example, in the model of Ref. [19], there are two massless mode solutions

²³The existence of a physical massless mode of B means that the radius is unstabilized.

in B . However, one of them can be gauged away within the Newton gauge and the other is forbidden by the boundary condition. These massless mode solutions are related to $\xi_y^\pm(x)$ through the boundary condition (B.10). Hence, the absence of the massless mode means that we can set $\xi_y^\pm = 0$ within the Newton gauge. Namely, $y^\pm(x)$ can be taken as constants. (See Eq.(B.3).) Note that this argument holds only *at the linearized level*. Including the higher order in the discussion, the boundaries cannot be expressed by $y = \text{constant}$ in the Newton gauge any more.

C Superfields and gauge fixing

Here, we will collect explicit forms of the $\mathcal{N} = 1$ superfields in terms of the superconformal notation of Ref.[14, 18].

C.1 5D superfields

A vector multiplet \mathcal{V}^I consists of

$$M^I, \quad W_\mu^I, \quad \Omega^{Ii}, \quad Y^{I(r)}, \quad (\text{C.1})$$

which are a gauge scalar, a gauge field, a gaugino and an auxiliary field, respectively. The indices $i = 1, 2$ and $r = 1, 2, 3$ are doublet and triplet indices for $SU(2)_U$. From this multiplet, we can define the following vector and chiral superfields.

$$\begin{aligned} V^I &\equiv \theta \sigma^m \bar{\theta} W_m^I + i \theta^2 \bar{\theta} \bar{\lambda}^I - i \bar{\theta}^2 \theta \lambda^I + \frac{1}{2} \theta^2 \bar{\theta}^2 D^I, \\ \Phi_S^I &\equiv \varphi_S^I - \theta \chi_S^I - \theta^2 \mathcal{F}_S^I, \end{aligned} \quad (\text{C.2})$$

where

$$\begin{aligned} \lambda^I &\equiv 2e^{\frac{3}{2}F} \Omega_R^{I1}, \\ D^I &\equiv -e^{2F} \left\{ G^{-1} \partial_y M^I - 2Y^{I(3)} + G^{-1} \dot{F} M^I \right\}, \\ \varphi_S^I &\equiv \frac{1}{2} (W_y^I + i G M^I), \\ \chi_S^I &\equiv -2e^{\frac{F}{2}} G \Omega_R^{I2}, \\ \mathcal{F}_S^I &\equiv -ie^F G (Y^{I(1)} + i Y^{I(2)}). \end{aligned} \quad (\text{C.3})$$

The hypermultiplets consist of complex scalars \mathcal{A}_i^α , spinors ζ^α and auxiliary fields \mathcal{F}_i^α . They carry a $USp(2, 2n_H)$ index α ($\alpha = 1, 2, \dots, 2n_H + 2$) on which the gauge group can act. These are split into $n_H + 1$ hypermultiplets as

$$\mathcal{H}^{\hat{\alpha}} = (\mathcal{A}_i^{2\hat{\alpha}+1}, \mathcal{A}_i^{2\hat{\alpha}+2}, \zeta^{2\hat{\alpha}+1}, \zeta^{2\hat{\alpha}+2}, \mathcal{F}_i^{2\hat{\alpha}+1}, \mathcal{F}_i^{2\hat{\alpha}+2}). \quad (\text{C.4})$$

From these multiplets, we can define the following chiral superfields.

$$\Phi^\alpha \equiv \varphi^\alpha - \theta \chi^\alpha - \theta^2 \mathcal{F}^\alpha, \quad (\text{C.5})$$

where

$$\begin{aligned}
\varphi^\alpha &\equiv \mathcal{A}_2^\alpha, \\
\chi^\alpha &\equiv -2ie^{\frac{F}{2}}\zeta_R^\alpha, \\
\mathcal{F}^\alpha &\equiv e^F G^{-1} \left\{ \partial_y \mathcal{A}_1^\alpha + i \left(G + \frac{iW_y^0}{M^0} \right) \tilde{\mathcal{F}}_1^\alpha - (W_y^I - iGM^I) (gt_I)^\alpha{}_\beta \mathcal{A}_1^\beta + \frac{3}{2} \dot{F} \mathcal{A}_1^\alpha \right\}.
\end{aligned} \tag{C.6}$$

In the last expression, $\tilde{\mathcal{F}}_1^\alpha$ are defined as

$$\tilde{\mathcal{F}}_1^\alpha \equiv \mathcal{F}_1^\alpha - M^0 (gt_0)^\alpha{}_\beta \mathcal{A}_1^\beta. \tag{C.7}$$

Since we treat the radion supermultiplet separately, we have not included the dependence on the $SU(2)_U$ gauge field $V_y^{(r)}$, which corresponds to an auxiliary field of the radion multiplet, in the above definitions of the superfields.

C.2 4D superfields

We can construct superfields also from 4D superconformal multiplets. From a vector multiplet $(B_m^A, \lambda^A, \bar{\lambda}^A, D^A)$, we can obtain the following vector superfield.

$$V^A \equiv \theta \sigma^{\underline{m}} \bar{\theta} B_m^A + ie^{\frac{3}{2}F(y^*)} \theta^2 \bar{\theta} \bar{\lambda}^A - ie^{\frac{3}{2}F(y^*)} \bar{\theta}^2 \theta \lambda^A + \frac{1}{2} e^{2F(y^*)} \theta^2 \bar{\theta}^2 D^A, \tag{C.8}$$

where $y^* = 0, \pi R$ are the brane locations. The corresponding superfield strength is defined as

$$\mathcal{W}_\alpha^A \equiv -\frac{1}{4} \bar{D}^2 D_\alpha V^A. \tag{C.9}$$

From a chiral multiplet $(s^a, \chi_s^a, \mathcal{F}_s^a)$, we can construct the following chiral superfield.

$$S^a \equiv s^a - e^{\frac{F}{2}(y^*)} \theta \chi_s^a - e^{F(y^*)} \theta^2 \mathcal{F}_s^a. \tag{C.10}$$

The warp factors in the above definitions come from the induced metric on the boundaries.

C.3 Superconformal gauge fixing

The gauge fixing conditions for the extraneous superconformal symmetries, *i.e.*, the dilatation \mathbf{D} , $SU(2)_U$, the conformal supersymmetry \mathbf{S} are as follows.²⁴

The \mathbf{D} -gauge is fixed by

$$\begin{aligned}
\mathcal{N} &\equiv C_{IJK} M^I M^J M^K = M_5^3, \\
\mathcal{A}_i^\alpha d_\alpha{}^\beta \mathcal{A}_\beta^i &= 2 \left\{ -\sum_{a=1}^2 |\mathcal{A}_2^a|^2 + \sum_{\underline{\alpha}=3}^{2n_H+2} |\mathcal{A}_2^{\underline{\alpha}}|^2 \right\} = -2M_5^3,
\end{aligned} \tag{C.11}$$

²⁴The special conformal transformation \mathbf{K} is already fixed in our superspace formalism [16].

where \mathcal{N} is called the norm function, and $SU(2)_U$ is fixed by the condition

$$\mathcal{A}_i^\alpha \propto \delta_i^\alpha, \quad (\alpha = 1, 2). \quad (\text{C.12})$$

The \mathbf{S} -gauge is fixed by

$$\begin{aligned} \mathcal{N}_I \Omega^{Ii} &= 0, \\ \mathcal{A}_i^\alpha d_\alpha^\beta \zeta_\beta &= 0, \end{aligned} \quad (\text{C.13})$$

where $\mathcal{N}_I \equiv \partial \mathcal{N} / \partial M^I$.

References

- [1] P. Hořava and E. Witten, *Nucl. Phys.* **B460** (1996) 506 [[hep-th/9510209](#)]; *Nucl. Phys.* **B475** (1996) 94 [[hep-th/9603142](#)].
- [2] A. Lukas, B.A. Ovrut, K.S. Stelle and D. Waldram, *Phys. Rev.* **D59** (1999) 086001 [[hep-th/9803235](#)]; *Nucl. Phys.* **B552** (1999) 246 [[hep-th/9806051](#)].
- [3] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83** (1999) 3370 [[hep-ph/9905221](#)].
- [4] W.D. Goldberger and M.B. Wise, *Phys. Lett.* **B475** (2000) 275 [[hep-ph/9911457](#)].
- [5] C. Csáki, M. Graesser, L. Randall and J. Terning, *Phys. Rev.* **D62** (2000) 045015 [[hep-ph/9911406](#)].
- [6] J. Garriga and T. Tanaka, *Phys. Rev. Lett.* **84** (2000) 2778 [[hep-th/9911055](#)].
- [7] C. Charmousis, R. Gregory, and V.A. Rubakov, *Phys. Rev.* **D62** (2000) 067505 [[hep-th/9912160](#)].
- [8] J. Bagger, D. Nemeschansky and R. Zhang, *JHEP* **0108** (2001) 057 [[hep-th/0012163](#)].
- [9] W.D. Goldberger and M.B. Wise, *Phys. Rev. Lett.* **83** (1999) 4922 [[hep-ph/9907447](#)].
- [10] C. Csáki, M.L. Graesser, and G.D. Kribs, *Phys. Rev.* **D63** (2001) 065002 [[hep-th/0008151](#)].
- [11] R. Altendorfer, J. Bagger and D. Nemeschansky, *Phys. Rev.* **D63** (2001) 125025 [[hep-th/0003117](#)]; T. Gherghetta and A. Pomarol, *Nucl. Phys.* **B586** (2000) 141 [[hep-ph/0003129](#)]; A. Falkowski, Z. Lalak and S. Pokorski, *Phys. Lett.* **B491** (2000) 172 [[hep-th/0004093](#)]; E. Bergshoeff, R. Kallosh and A. Van Proeyen, *JHEP* **0010** (2000) 033 [[hep-th/0007044](#)]; M. Zucker, *Phys. Rev.* **D64** (2001) 024024 [[hep-th/0009083](#)].
- [12] W.D. Linch III, M.A. Luty and J. Phillips, *Phys. Rev.* **D68** (2003) 025008 [[hep-th/0209060](#)].

- [13] D. Marti and A. Pomarol, *Phys. Rev.* **D64** (2001) 105025 [[hep-th/0106256](#)].
- [14] T. Kugo and K. Ohashi, *Prog. Theor. Phys.* **108** (2002) 203 [[hep-th/0203276](#)].
- [15] F. Paccetti Correia, M.G. Schmidt and Z. Tavartkiladze, [hep-th/0408138](#).
- [16] H. Abe and Y. Sakamura, *JHEP* **0410** (2004) 013 [[hep-th/0408224](#)].
- [17] N. Maru and N. Okada, *Phys. Rev.* **D70** (2004) 025002 [[hep-th/0312148](#)].
- [18] T. Kugo and K. Ohashi, *Prog. Theor. Phys.* **105** (2001) 323 [[hep-ph/0010288](#)].
- [19] M. Eto, N. Maru, and N. Sakai, *Nucl. Phys.* **B673** (2003) 98 [[hep-th/0307206](#)].
- [20] T. Tanaka and X. Montes, *Nucl. Phys.* **B582** (2000) 259 [[hep-th/0001092](#)].
- [21] B. de Wit, P.G. Lauwers and A. Van Proeyen, *Nucl. Phys.* **B255** (1985) 569.
- [22] T. Kugo and S. Uehara, *Nucl. Phys.* **B226** (1983) 49; *Prog. Theor. Phys.* **73** (1985) 235.
- [23] J. Bagger and M. Redi, *JHEP* **0404** (2004) 031 [[hep-th/0312220](#)].
- [24] M.A. Luty and R. Sundrum, *Phys. Rev.* **D64** (2001) 065012 [[hep-th/0012158](#)].
- [25] L. Hall, J. Lykken and S. Weinberg, *Phys. Rev.* **D27** (1983) 2359; A. Pomarol and S. Dimopoulos, *Nucl. Phys.* **B453** (1995) 83 [[hep-ph/9505302](#)].
- [26] J. Scherk and J.H. Schwarz, *Phys. Lett.* **B82** (1979) 60.